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ABSTRACT

This booklet contains copies of the two major presentations at the SMSG Conference on Future Responsibilities for School Mathematics: "A History of Attempts to Improve School Mathematics in the United States" (Phillip S. Jones), and "The Future of Mathematics Education in the United States" (Marshall H. Stone). A summary of the discussions following these two addresses is included, along with a listing of the final recommendations for general operational procedures, research and development of curricula, implementation, and evaluation and follow-up studies. (DT)

SCHOOL MATHEMATICS STUDY GROUP

Report of

A CONFERENCE ON FUTURE RESPONSIBILITIES FOR SCHOOL MATHEMATICS

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PREFACE

When the School Mathematics Study Group came into being on March 1, 1958, it was not clear how long it would take to accomplish its objectives. A few years of experience, however, indicated clearly enough that the general kinds of activities undertaken by SMSG, in particular the close collaboration between classroom teachers and research mathematicians, ought to be continued indefinitely. The completion of several major projects gave evidence of the value of such collaboration; in the course of completing them the need for several new projects developed.

For this reason a Conference on Future Responsibilities for School Mathematics was held on February 24-25, 1961. The purpose of the conference was to consider ways of continuing the work which SMSG had begun. The nearly fifty participants represented all parts of the country and all parts of the mathematical profession, as well as various degrees of interest in mathematics education, ranging from full-time participation in curriculum projects to passive but interested observation from the sidelines. This is a report of that conference.

The conference program began with two major presentations: "A History of Attempts to Improve School Mathematics in the United States," by Professor P. S. Jones, University of Michigan, and "The Future of Mathematics Education in the United States," by Professor M. H. Stone, University of Chicago. The former presentation was an address on the history, both ancient and modern, of attempts to improve the school mathematics curriculum in this country. The latter was a presentation of the personal thoughts on the future of school mathematics in this country by a distinguished mathematician who for a long time has had a deep interest in problems of mathematics education.

A digest of the discussion which followed the addresses is in this report.

The conference participants submitted specific suggestions for appropriate procedures and mechanisms for attacking problems were brought to light.

A committee arranged and edited the suggestions which were then discussed the second day of the conference. The final series of discussions resulted in a set of recommendations which were adopted unanimously by the participants in the conference. It was also recommended that the Advisory Committee, which sets the basic policy for the School Mathematics Study Group, be given a more formal structure. This recommendation was embodied in the Bylaws of SMSG which were adopted by the Advisory Committee (now renamed the Advisory Board) in October 1961.

TABLE OF CONTENTS

A HISTORY OF ATTEMPTS TO IMPROVE SCHOOL MATHEMATICS IN THE UNITED STATES

Phillip S. Jones

My major objective here is to provide "hindsight" and historical background that can be useful during consideration of plans for the future. It is tremendously important that we do consider now, quickly and earnestly, what sorts of things can be done to extend and consolidate not merely the changes in mathematics education, but also what can be done to maintain a positive rate of change in the years to come.

I accepted Professor Begle's invitation to do this with alacrity because I thought it was important and also because making a survey of what have been the causes and effects of proposals and reforms has been a project interesting me for some time. As with all other hasty decisions, or most of them, I have regretted this acceptance. There are still so many unanswered questions, so many things that I thought of doing and that I have not finished doing! However, here is the current status of my unfinished business.

It would be fun to look at the journals and survey their comments on new proposals and reviews of new texts at each period. It would be fun to look at the textbooks prior to, during, and after the period of a major committee report or curriculum proposal to see what reception they had and what changes actually took place. I tried all of these but the task is only partially complete.

One of my friends had a twinkle in his eye, I believe, when he said, "Well, you know if you take just a little bit from somebody else you may be accused of plagiarism but if you take a great deal, large segments, you can call it historical research and get away with it." At any rate, I want to acknowledge my sources in a general way at the outset and then not attempt to document each item later. I began my survey of these past programs with the historical chapters and the notes and references in C. H. Butler and Lynwood Wren's The Teaching of Secondary School Mathematics, and the article on "Mathematics" by Theodore Breslich, of the University of Chicago, in the

book called A Half Century of Science and Mathematics Teaching, which is a semi-centennial volume put out by the Central Association of Science and Mathematics Teachers. I did not draw all my information from those two sources but I began there and I want to give them credit. I have also looked at many of the original reports, but I don't think that the detailed documentation of all of this is significant here. I also thought it would be fun to write to people, both here and abroad, to get their reaction to some questions. I did a little of this as will appear later.

I have largely limited myself to developments in this country and, in general, to recent developments. I resisted the temptation to put in many side issues and sidelights that were fun to note, but I couldn't resist completely. One of the items I could not resist inserting was some of the prehistory of mathematics in American education, because to me this is part of our fascinating story.

You will note that the first book with mathematical content printed in the Americas was printed in Mexico City in 1556. It was Juan Diez Freyle, Summario compendioso --- de plata y ore, which had for its major concern the treatment of the value and different coinages of different purities of gold and silver ore. How consistent this is with the Spanish American, South American, Central American, period of conquest and exploration! Written by a missionary, a member of a religious order, but dealing with the value of gold ore!

It is rather interesting, however, that this first book with mathematical content aimed at this very practical problem of arithmetic did also contain some algebra. There was no apparent very close connection with the rest of the book or its purpose, but there was some algebra in there.

I will not elaborate on the other early works, but in the Western Hemisphere prior to 1700 there were seven Mexican and four Peruvian books. (We so often lose sight of the fact that Central and South America are much older than we in terms of printing, university instruction, etc.!) These books, though they contained mathematics, in their titles and major content dealt with military matters, e.g., how you set up a military camp, with

navigation, with translating ore into monies, with the calendar and the determination of fast (feast) days and religious celebrations.

I might pause just because it might interest you to note the title of the book by Pedro de Paz, published in Mexico City in 1623, Arte para aprender toto el menor del arithmetica sin maestro, "without a Master." In other words "home" and "self-study" was well begun in 1623.

The first book with mathematical content printed in North America was John Hill's, The Young Secretary's Assistant, which appeared in Boston in 1703. As its title implies, it dealt with how to write a business letter, how to keep books, etc. The secretary to the man who was sending out ships to trade, for instance between the Indies and Europe, had to know a little bit of arithmetic, of weights, measures and coinages and how to translate them into different currencies. This Young Secretary's Assistant, which contains some mathematics, was actually in a sense pirated. Perhaps it isn't fair to say that. Customs and laws dealing with copyrights were much different then. Most of our early colonial books were just reprints of English, later on French, books. The first North American (excepting Mexico) written by a person in this country was Isaac Greenwood's Arithmetic, Vulgar and Decimal, in 1729.

The first book with any substantial geometric content was Hawney's Complete Measurer, printed in 1801. It dealt largely with simple surveying, weights, measures, cordwood, barrels of wine, etc.

The point I would make in this is that mathematics was very late in arriving in North America and it was largely brought to us by foreigners, if you include the English in that category. It not only was late in arriving, brought from abroad, but very largely stressed practical uses.

Now turning to the early schools, and their programs. The predecessor of our secondary school was the Latin Grammar School. I believe the Boston Latin Grammar School was established in 1635. It was a private school for a middle class clientele, preparing for the professions, ministry and law, medicine and college.

A little later we have the beginning of the academies. Benjamin Franklin founded one in 1751, but they did not really develop until sometime later, flourishing from 1787 to 1870. They were a sort of a common man's version of the Latin Grammar School. That is, they still were private, but they put a little more stress on preparation for commercial enterprises, business, etc.

The mathematics curriculum of these grammar schools and academices I would classify under three headings: purposes, presentation, content. There were two purposes: (1) practicality, the uses of arithmetic and geometry in mensuration, calculating money in different currencies, etc., (2) "mental discipline," the idea was that it's good for your mental muscles to wrestle with Latin and arithmetic.

Their methods of presentation involved the presentation of many rules, straightforward statements, often in italics, to be memorized, a worked example or so for each, and then problems to be done.

The content of the mathematics curriculum of the grammar schools and academies may be well illustrated by a glance at a typical text, Pike's Arithmetic, one of the most popular. The second edition, 1797, had 396 pages of arithmetic, but there is much there which many of us wouldn't recognize easily, such as the rule of three and other terminology which is no longer familiar. It also contained four pages of geometry, 11 pages of trigonometry, 46 pages of mensuration, 33 pages of algebra and 10 pages devoted to conics.

I might pause for just a moment to tell you a little bit about college requirements and how they tied in. Harvard required arithmetic for admission in 1807. Algebra was required for admission into Harvard in 1820. Yale followed Harvard in 1847 and Princeton in 1848, requiring algebra for admission. In geometry Yale led all the rest. In 1865 they required geometry. Princeton, Michigan, and Cornell had geometry as an entrance requirement in 1868, Harvard in 1870, but when Harvard did do it, they required logarithms as well as geometry.

Of course none of these periods or concepts are completely disjointed. Educational programs and institutions grow and develop, they neither begin

nor end at a given instant and schools with different patterns coexist. However, although the earliest public high school is usually dated as 1821, the system of public high schools as we tend to think of it really developed from 1875 to 1900. You see by all this that mathematics education, like mathematics research, has developed rather slowly and recently in this country.

Turning to the period of the development of the public high schools and their immediate forerunners, it's a little interesting to look at some old textbooks. The characteristics I mentioned above can easily be seen. Here is a quotation from Daboll's <u>Schoolmasters Assistant</u>, (1799). The discussion of addition of "vulgar" fractions begins on page 163 as follows:

> "Reduce compound fractions to single ones, mixed numbers to improper fractions; and all of them to the least common denominator (by Case VI, Rule II) then the sum of the numerators written over the common denominator will be the sum of the fractions required."

That was it. That was the rule. This statement was followed by one worked example and fourteen problems.

It is interesting that decimal fractions, which of course had originally developed much later than common fractions, were discussed in Daboll on page 78, because of their use in dealing with dollars and cents. How do you teach intelligently about decimal fractions without first teaching common fractions? You do it by just telling them rules. Of course the rules for operating with decimal fractions are very much like those for integers. Hence, at that time and under that philosophy it was sensible and easy to teach decimal fractions on page 78 and vulgar fractions on page 163. Other topics in Daboll's book were interest, discount, currencies, exchange, false position, annuities, tare, tret, practice.

Now let's turn to Quackenbos' <u>Elementary</u> <u>Arithmetic</u>. The copy on the shelf in my study was copyrighted 1866 but it was used by my father and has his name in it and the date 1892. Page 78 begins the discussion of fractions as follows: "Halves and halves, thirds and thirds, etc. can be added just as we add pears and pears, dollars and dollars." (Incidentally, Daboll used apples instead of pears and did not include dollars.) That was paragraph 130

and there was just a bit more discussion in it of adding fractions with common denominators.

Paragraph 131 on the same page reads, "Halves, thirds, etc. cannot be thus directly added any more than we can add pears and dollars." This was followed by an example of how to add halves and thirds. Following the example came a rule. Following the rule were "exercises for the slate." And that was it! On two small pages he covered adding fractions with and without common denominators, gave examples and a set of problems.

My last example of this series is admittedly thrown in for the fun of it. This is from The Poetical Geography by George Van Waters. This Poetical Geography contained in an appendix the rules of arithmetic, in verse. I will, however, warn you in advance that this poem is about square root. I picked it because I thought it was just a little bit more fun than fractions.

> "Divide into periods of two figures each,
> The number you know as the pedagogues teach.
> In the left hand period find the greatest square,
> Which from it subtract, and to what remains there
> Bring the next period down for a dividend (fair).
> Place the root of the square at the right hand of all,
> And two times the root a divisor we call."
> Etc.

The amusing thing about this, by the way, is its use of double columns. The right hand column gives you the same thing in prose.

Although this procedure of stating rules, doing examples and giving exercises was typical, there were, of course, persons trying to do other things.

Warren Colburn wrote Intellectual Arithmetic Upon the Inductive Method of Instruction, in 1821. I took down from my shelf his Introduction to Algebra Upon the Inductive Method of Instruction, which in the copy I have, is dated 1830. There may have been an earlier edition. I am quoting from its preface:

> "The first object of the author of the following treatise has been to make the transition from arithmetic to algebra

There is a strong need for scientific facts from psychology concerning topics such as the development of the mind, concept formation, and attitude formation. We, therefore, have the problem of working, not only with the curriculum, but working with the adjustment of the curriculum to psychological facts.

There is a need to emphasize more than we are at the present work at the elementary school level. This is the point of origin of our mathematicians. In addition there is the task of giving a general preparation for all students, not merely the college bound students.

The development of modern technology seems already to be making a very marked shift in the nature of employment by industry. Automation alone is having an effect. An education which involves certain kinds of mathematics that have an application to such things as electrical circuitry may be necessary.

There is also the question of the coordination of mathematics with other school subjects. This also means that mathematics cannot subscribe to the suggestion or demands of any one particular group. There should be some joint studies of the fundamental elements of the problem. We must get to the fundamental ideas that are centrally important and decide somehow how we can answer the questions about these things. A corollary to this is the preparation of teachers. Again it is the problem of developing a clear picture of what it is we want to teach and then giving the best preparation we can give for teaching this kind of material. It appears sound that a teacher should know more than they will teach at a particular level. This standard appears universally applicable.

There is a need to pay attention to the content of the profession of teaching.

The main points apart from the central problem of working out further curricular measures include:

1. A special effort with the elementary school level.
2. Attention to the contribution psychology can make.

> as gradual as possible. The book therefore commences with practical questions in simple equations such as the learner might readily solve without the aid of algebra..."
>
> "The most simple combinations are given first, then those which are more difficult. The learner is expected to derive most of his knowledge by solving the examples himself;..."
>
> "In fact, explanations rather embarrass than aid the learner because he is apt to trust too much to the man, and neglect to employ his own power; and because the explanation is not made in the way that would naturally suggest itself to him if he were left to examine the subject by himself...This method besides giving the learner confidence,...is much more interesting because he seems to be constantly making new discoveries..."

Skipping to another paragraph:

> "In the ninth article the learner is taught to generalize particular cases and to form rules....The learner should solve every question. When the learner is directed to turn back and to do in a new way, something he has done before, let him not fail to do it, for it will be necessary for his future progress..."
>
> "The author has heard it objected to his arithmetics by some, that they were too easy,...if they are too easy it is the fault of the subject, not of the book.... The intellects of his scholars are more exercised in studying them than in studying the most difficult treatise he can put in their hands."

Pike and Daboll with their rule oriented approach were more typical and popular than Colburn's inductive approach. However, as time passed we began to get texts with a blending of the inductive and the rule approach. Although the aims of both were largely practical use and mental discipline, there developed a concern for programming a learning process so as to gain the student's interest and increase his confidence, by leading him to discover and generalize.

In this connection I would like to add one more note on the history of the "discovery" teaching process. This was called to my attention by William

Fitzgerald, a student of mine who is now teaching at Eastern Michigan University. He brought me a copy of The Alexander-Dewey Arithmetic published in 1920. On its title page John Dewey, editor of the series, saw printed the following quotation attributed to M. Laisant in L'Enseignement Mathématique:

> "The problem is always the same: to interest the pupil, to induce research, to give him the notion continually, the illusion if you please, that he is discovering for himself, that which is being taught him."

I shall now stop, as I shall repeatedly, to try to draw what seem to be, if not "morals," then concepts and generalizations that should be emphasized as implicit in the specific quotations and examples I have cited. A first is, I suppose, the same question for mathematics education which is raised so often elsewhere, "Is there really anything new under the sun?" I think there is. But I also think it is also illuminating to see to what extent the things we are talking and doing now represent a continuation of ideas that have been proposed previously. Further, such considerations bring us to another question which is critical for our discussion here: "If similar things have been proposed and failed or been partially abandoned, how much acceptance did they get and why did they not gain more?"

Another pair of questions is to what extent do psychological and philosophical considerations and concepts actually effect education? I believe we have shown that they do but that the means through which they do it and the rate of the resulting change is quite uncertain. I find these difficult questions to answer. In our examples, concerns for interest, motivation, discovery, proceeding from concrete to abstract and from specialization to generalization to application were expressed and did seem slowly to be followed by some changes, but only slowly and with an uncertainty as to how the change was wrought. Perhaps a problem for us here is to seek ways for both identifying developing philosophical and psychological principles of importance and then accelerating their acceptance and interpretation in classrooms.

It is easy to cite even more recent examples of the above problem.

G. D. Birkhoff and Ralph Beatley's Basic Geometry was a publishing failure in the 1930's. It is now being reprinted and has influenced several recent texts. Even earlier, G. B. Halsted of Texas wrote Rational Geometry, an attempt to use Hilbert's postulates in a high school geometry. It died very rapidly, but we have completed some sort of a cycle, and again we have new texts based on Hilbert.

As a step toward an examination of methods for translating philosophical convictions into school room reforms, I'd like to interpolate a little bit about mathematics in England. There are several justifications for this in addition to the mere fact that it is interesting. Any history of mathematics education in this country must mention E. H. Moore, and his influence in the early days of this century. Moore was much influenced by and often cited John Perry. Englishman John Perry's views were, of course, developed in the context of English secondary school and college mathematics and relationships between them. So it is not wholly foreign to our major topic here to talk a little about England.

One of the latest pamphlets from the British Ministry of Education, Teaching Mathematics in the Secondary Schools, states, "Mathematics hardly existed in schools until the time of Queen Victoria, whose reign began in 1837." It is interesting to compare this with our earlier statements about American schools.

In 1861 The Royal Commission on Nine Public Schools said, "The chief honors and prizes in our public schools go for the Classics which are to the great number of the boys, intrinsically more attractive than Mathematics... but mathematics, at least, has established a title of respect as an instrument of mental discipline." Another quotation from this same group refers to a situation which has been much more significant in England than in this country but it may become increasingly significant in this country. This is the relationship between the secondary schools and the college entrance examinations, or as they are called in this report, "the external examinations." The quotation is, "The introduction of mathematics as an integral

part of the school curriculum coincided with the establishment of external examinations. This circumstance tended to set a pattern of mathematics education before much thought had been given to the matter." The curriculum in England at that time included algebra, geometry, surveying, navigation, mensuration and trigonometry.

Following this, toward the end of the 19th century, (in England, as in this country later) student failures in mathematics and Latin increased and colleges complained of the poor preparation of their students.

These quotations and facts point out the following "morals:" (1) College criticisms have motivated establishing and changing external examinations which in turn have very significant effects on the school curriculum. (2) The changing nature of the school population has also been a factor in the design of school programs. (3) Practical and intellectual objectives have always had a place in our school programs, but have varied in their relative importance.

Now turning to what I suppose maybe could have been my sole topic, though I think not, the reform movements in the United States during the first half of the twentieth century. I have appended a list of these. It is not complete, but I think it is nearly so and reasonably representative.

The first two of these actually took place in the last decade of the nineteenth century. The Committee of Ten on the Secondary School Syllabus of 1894 had a Subcommittee on Mathematics. This Committee arose out of the complaints of the colleges of the preparation of their students, and the complaints of the high school administrators of the number of failures in mathematics and Latin, especially. The Sucbommittee on Mathematics recommended (a) the introduction of "concrete" (intuitive) geometry with concrete examples, (b) the introduction of algebra at 14 years (8th or 9th grade), (c) the integration of plane and solid geometry, and (d) parallel instruction in both algebra and geometry following the introductory year. The idea of a parallel program as well as item (c) suggests a concern for reducing the compartmentalization of our mathematics curriculum. This too is a modern concern.

In 1899, the National Education Association set up a committee on college entrance requirements. I believe that the American Mathematical Society assisted in setting up their mathematics committee and appointed one or two of the delegates.

Their recommendations were very much like the recommendations of the Committee of Ten. The seventh grade ought to include concrete geometry and introductory algebra. The eighth grade should include demonstrative geometry and formal algebra. The ninth and tenth should continue with geometry and algebra taught at the same time. The eleventh grade was to be solid geometry and trigonometry, while the twelfth was to include advanced algebra and review. Several of these recommendations seem quite modern.

In my list of movements I interpolated in 1901 and 1902 John H. Perry and E. H. Moore without giving them numbers because obviously they are not movements. However, their contributions and presence at that time were such that I think that they should be noted.

John Perry spoke in 1901 to the Mathematical Association in England, and out of his speech came a committee that reported a year later. Its report did make significant changes in the examinations and syllabus for geometry in England in the direction of allowing more flexibility in answers rather than reproduction of Euclid. I'm not going to go into that, but among Perry's own emphases were: the use of the laboratory method of teaching, stress on the correlation between mathematics and science, acceptance of some theorems without proof, and discussion of practical uses.

I cannot resist the temptation to read to you a quotation from this discussion in 1901. You will be interested that among persons who contributed to this discussion, either written comments or through their own presence, were Mrs. Boole, Miss Scott from Bryn Mawr, Horace Lamb, David Eugene Smith, Lord Kelvin, and Oliver Heaviside. Here is the quotation from Heaviside:

> "Boys are not philosophers and logicians...Now the prevalent idea of mathematical works is that you

> must understand the reason first before you practice. This is fuss and fiddlesticks...I know mathematical processes that I have used with success for a very long time of which neither I nor anyone else understands the scholastic logic... There is something wanting, no matter how logical people may pretend... Geometry should be entirely observational and experimental at first..."

I hardly need note that this attitude has its modern advocates.

Other European mathematicians were concerned for reform in the schools: Klein, Tannery, Borel, Nunn. I wrote my friend Lucas N. H. Bunt of the University of Utrecht for his analysis of European reforms. He wrote:

> "It took years and years before the results of Klein's suggestions in this direction (the use of transformations in geometry) were seen in high school teaching, and still at the moment it is not at all clear whether the changes which many German (and Russian) mathematics school books have gone through will be permanent. As far as they really follow the directions given by Klein, they are unreadable...Why?... Klein suggested something which sounded reasonable and attractive as long as it was expressed in vague terms. But he did not try to put his proposals in form applicable in the classroom... If Klein had tried to write out the details about his idea of teaching transformations as a basis for studying a first course in geometry, he would have seen immediately that they were unrealistic."

Whether or not this is true (and I think it is) my purpose in presenting these quotations and these anecdotes is not merely to point out again that reform in mathematics education is not a new idea, but also to underline the importance of new texts and of getting down to specifics both in writing and in experimental teaching.

Birkhoff and Beatley and Halsted wrote texts. It is not a matter of record why they failed, but probable reasons are lack of adequate teaching experience followed promptly by revisions. Another probable pair of reasons is that there was no general sentiment for change at the time nor any strong committee or group calling for a change or endorsing their works.

Notice also the recurring confusion with respect to rigor vs. intuition,

with respect to rule vs. meaning. How do we obtain for each of these a proper recognition of its role in teaching and exposition, and, I think, even in research and development? I don't know the answer but at least we can be sure it's a time-honored problem.

In 1911 and 1918 the International Commission on the Teaching of Mathematics had some influence in this country. I think its influence was usually secondhand. I doubt that it influenced textbook writers or teachers directly, but it did influence later American commissions and writers. Several of its recommendations sound familiar to anyone who has read the 1923 report or even more recent reports. Some were: omit long lists of definitions at the beginning of courses and "obvious" proofs, transfer difficult algebra to the tenth grade or later, avoid complex manipulations, emphasize the equation more, use problems from life and physics, modify the teaching in terms of recent psychological advances with regard to formal discipline.

These recommendations marked a sort of swing of the educational pendulum, from formal and disciplinary teaching at an earlier age toward "postponement," less formal teaching and practicality and concreteness. Some of this was motivated by sound psychology and pedagogy, some of it represented the effects of the changing school population, and some of it represented misinterpretations of popular psychology and educational philosophy.

In 1911 a Committee of Fifteen on the Geometry Syllabus was appointed. They advocated the use of concrete exercises and graded originals, but also called for the recognition in teaching of the logical structure of geometry and the nature of undefined terms, the introduction of definitions as needed, the use of some informal proofs, and the exclusion of limits and incommensurables.

The Mathematics Association of America and the National Council of Teachers of Mathematics were being founded in the same period as witnessed the rise of the junior high school, 1915 - 1920.

In 1923 the National Committee on Mathematical Requirements of the Mathe-

matical Association of America brought in its famous report after seven years of work. It classified the objectives of mathematical instruction as practical, disciplinary and cultural, and stressed the role of the function concept in the teaching of mathematics. Most texts for years thereafter cited this report in their prefaces and honored the then current interpretation of the "function concept" by stressing the interrelationships between tables, graphs, formulas, and equations.

However, perhaps the one committee recommendation which more than all others had an effect was their stress on general mathematics, especially at the junior high school level. To explain this I must take a little time out to talk about general mathematics.

General mathematics as then advocated by many people, mathematicians and others, was considered to be a reorganization of good, sound, steadily advancing algebra, geometry, trigonometry, and introductory statistics. The idea was to reduce the compartmentalization of mathematics, to introduce more stress on interrelationships. Actually, however, general mathematics was very hard to sell. The one place where it turned out to be easy to sell was in the seventh and eighth grade junior high school program because the junior high was just developing and because general mathematics was consistent with the philosophy of the junior high school.

The role of the junior high school was conceived to be exploratory, to help students to see a wide range of areas of interest, to evaluate for themselves what they wanted to do, to provide for them vocational guidance. Everybody was going through junior high school so the eighth grade was regarded as almost terminal. It was more terminal in those days than now. This newly risen junior high school and its educational philosophy were a perfect setting for the introduction of general mathematics in place of arithmetic.

Another reason for the success of the general mathematics recommendations of this group in grades seven and eight was the fact that texts were produced

by persons associated with the committee and others. Schorling and Clark, Reeve, Breslich and others had been already at work on such materials.

Above the eighth grade, however, general mathematics failed, at least in the sense originally intended. It became associated especially at the ninth grade level, with low level mathematics, poor mathematics. Those ninth graders who couldn't do algebra studied general mathematics. Thus the term "general mathematics" has meant different things at different times and to different people, and rarely has it been the high level fused course which was envisioned by its original advocates. Only a few integrated or "fused" course books were prepared for the senior high school program and none were widely used.

The 1923 report did, however, advocate four years of mathematics in high school, but the elective courses it suggested, investment, surveying, navigation, descriptive and projective geometry, were never set up. Incidentally, this 1923 Committee had a grant of money from a Rockefeller supported fund which enabled it to set up a central office with a staff of two people to carry on much of its work! However, it was accused of doing only half a job. Harold Rugg wrote in an evaluation of this report in The Mathematics Teacher for January 1924, "The National Committee, having large funds at its command (I believe its initial grant was for $25,000--it may have had more) employed the second type of service (mathematicians). It did not employ specialists in curriculum-making."

My list of reform projects and movements continues with the College Entrance Examination Board. It had been approached by the 1923 Committee which urged reduction in the emphasis on manipulation, testing for understanding and the abilities to draw conclusions and to apply mathematics as well as changes in content. It had made some modifications in its program and in its tests as a result of requests from this 1923 Committee. In 1935 the C. E. E. B. set up its Alpha, Beta, and Gamma examinations, to test persons going to college with different levels of mathematical training. Even in this country the importance of external testing must be recognized, but

the testing is itself partially responsive to curriculum changes.

Two groups reported in 1940. One had been appointed by the Progressive Education Association, and the other was the Joint Committee of the Mathematical Association of America and the National Council of Teachers of Mathematics appointed to study the place of mathematics in secondary education.

The first group expounded a long range philosophy. They stressed the importance of teaching problem solving and the nature of mathematical structure and wrote about data, approximation, function, operation, proof, and symbolism.

This report suffered some in its influence for being associated with the Progressive Education Association, I believe. There were many very good and rather modern ideas in the report.

The second group proposed a more detailed and specific curriculum for immediate adoption. It is also interesting to contrast it with the 1923 group. The 1923 Committee had one of the things we hear of as most typical of the present day situation with reference to reform in mathematics, the active participation of many members of the mathematical community. The membership of the 1923 Committee included: J. W. Young, A. R. Crathorne, C. M. Moore, E. H. Moore, D. E. Smith, H. W. Tyler. There were others but more than half of the group were people who came from college mathematics staffs.

The Joint Committee which reported in 1940 had the same number of representatives from the Association as from the National Council. The Association representatives were: K. P. Williams, A. A. Bennet, H. E. Buchanan, F. L. Griffin, C. A. Hutchinson, and H. F. MacNiesh and U. G. Mitchell.

The Joint Committee stressed individual differences. It recommended a two or more "track" curriculum. This was again a product of the time, the condition of the schools, and the condition of mathematics in the schools. They also wrote about transfer of training. The notion of transfer of training was for a time substantially discredited by psychologists. It is now beginning to be regarded again as an important educational element but with several theories, some new, to describe and explain it and the conditions

under which it takes place.

Our annotated history is now so near to the present time that more than a hasty listing of successive reforms and projects would be repetitious of the audience's own experiences.

Duke University began its famous summer institutes for mathematics teachers in 1941.

The A.A.A.S. Cooperative Committee on the Teaching of Science and Mathematics has functioned for many years.

The war brought on reports on "Pre-Induction Courses" and "Essential Mathematics for Minimum Army Needs," and lead to the Committee on Post War Plans of the National Council of Teachers of Mathematics. The major accomplishments of this latter Committee were the formulation of a definition of the minimum content of the mathematical education of every high school graduate and the formulation and support of some theses with respect to curriculum construction in mathematics. They took a strong stand against the abuses which had grown up of such psychological-philosophical curriculum determiners as "felt-need," "readiness," "postponement," "incidental learning," drill, and mathematics as a mere "tool" subject.

There were implications for mathematics education in the Harvard report of 1945, the reports of the President's Scientific Research Board (1947), and the President's Commission on Higher Education (1947). The strong pleas for the inclusion of some work in probability and statistics and for the reduction of the emphasis on solid geometry find their rebirth in recent times in the report <u>General Education in School and College</u> published by a group of eastern universities and preparatory schools in 1952.

Following this we have the development of the University of Illinois Committee on School Mathematics, The Advanced Placement Program, The Commission on Mathematics, the School Mathematics Study Group, and others.

Running through these comments you find the following factors often associated with successful innovations and recommendations and lacking where

proposals failed: the importance of "authority" and prestige, (note the role of the Association in England and of certain associations and schools in this country), the importance of texts, down-to-earth classroom materials (they aren't enough, but their lack will defeat a desirable recommendation), the importance of revised teacher training is not so clearly documented, but I believe its lack accounts for the failure of some projects with texts and others with strong authoritative support. External examinations have a rather clear effect, but the role of educational philosophies and psychologies is hard to assess. They influence administrators and teachers in other areas whose cooperation is essential. Today we have unprecedented sums of money and public interest and support. We can not expect these to endure indefinitely at the same level. We must make the most of our presently favorable position, but fairly, and then take action to guard ourselves against a growing lethargy and diversion of the attention of both the mathematical and mathematics-education communities. I believe we need a plan for regular and frequent reassessment of both the school mathematics program and the implications for it of the continuing developments in mathematics and its applications which I am sure will come.

THE FUTURE OF SCHOOL MATHEMATICS
IN THE UNITED STATES
(Abstract)

Marshall H. Stone

The purpose of this meeting is to consider the aims of mathematics education. Our objective is to provide the young people of America with the best possible mathematical education.

The volume of mathematical research that is reshaping mathematics is continually increasing. The consequences for high school, elementary school, and kindergarten are profound. The atmosphere of geometry at the research level and the nature of modern algebra are being reflected in the demands on what the secondary schools should do.

SMSG has been wise in not trying to do too much at first. But in looking to the future much more is going to have to be done. It has been wise to look first at the college bound student. Eventually, however, it will be necessary to conduct experiments with the various bodies of students.

The central problem is that of deciding what we should teach in mathematics. Satisfactory progress seems to be evolving at the graduate level and the problem is receiving a great deal of attention at the high school level. Not as much effort or imagination has been applied to either the college or elementary school levels. The core of the problem is analyzing the complete structure from top to bottom. Auxiliary questions arise as to the demarcation of the various levels of education--high school and college, college and graduate school, etc. Where do these divisions actually stand and how should the mathematics curriculum be fitted into this compartmentalization?

It is perfectly clear that, in any subject, as the discipline progresses the more advanced material has to be pushed down all the time in the scale. For instance, there appears to be a very strong movement to place the first course in calculus into the last year of the high school.

There exist also the fundamental psychological facts. With developing minds there are some things which perhaps you cannot do in certain stages.

There is a strong need for scientific facts from psychology concerning topics such as the development of the mind, concept formation, and attitude formation. We, therefore, have the problem of working, not only with the curriculum, but working with the adjustment of the curriculum to psychological facts.

There is a need to emphasize more than we are at the present work at the elementary school level. This is the point of origin of our mathematicians. In addition there is the task of giving a general preparation for all students, not merely the college bound students.

The development of modern technology seems already to be making a very marked shift in the nature of employment by industry. Automation alone is having an effect. An education which involves certain kinds of mathematics that have an application to such things as electrical circuitry may be necessary.

There is also the question of the coordination of mathematics with other school subjects. This also means that mathematics cannot subscribe to the suggestion or demands of any one particular group. There should be some joint studies of the fundamental elements of the problem. We must get to the fundamental ideas that are centrally important and decide somehow how we can answer the questions about these things. A corollary to this is the preparation of teachers. Again it is the problem of developing a clear picture of what it is we want to teach and then giving the best preparation we can give for teaching this kind of material. It appears sound that a teacher should know more than they will teach at a particular level. This standard appears universally applicable.

There is a need to pay attention to the content of the profession of teaching.

The main points apart from the central problem of working out further curricular measures include:

1. A special effort with the elementary school level.
2. Attention to the contribution psychology can make.

3. The study of high school mathematics for non-college bound students.
4. Coordination of mathematics teaching with the teaching of some of the applications of mathematics.
5. The preparation of teachers.

There is a need of some continuing mechanism for thinking about the central problem and the several tangential problems. It must also be remembered that the core of these problems is always an intellectual problem and not a practical problem. Fortunately, it appears that in the field of mathematics the various differences of opinion appear reconcilable. It would also appear helpful to think of our problems in terms of a few central concepts or ideas that we are trying to teach or attitudes that we are trying to convey.

The preparation of teachers must include a consideration of some attitudes toward subject matter.

A few phases that appear to indicate the main tendencies that mathematicians today apparently agree on include: the role of the function concept as a central theme, the structural concept in algebra, the Cartesian approach to get out of geometry into algebra as fast as possible, earlier introduction of vectors into geometry, the central notion of an axiomatic system, a continuing perplexity on how to handle the real numbers.

SUMMARY OF DISCUSSIONS FOLLOWING MAJOR ADDRESSES

Since the impact of sound recommendations is usually delayed, it would be well to create in the mathematical community a group which would have continuous existence, to promote steady curriculum change of a desirable character. Such a body should have rotating membership which will be sensitive to the rapid and continuing change in the science of mathematics. It should direct its efforts toward a well integrated curriculum for grades K-12, and in fact K-16. The importance of a coordinated program over the school and college years underlines the importance of full-time working groups (e.g., UICSM, Ball State, etc.) as well as the body suggested above.

The central problem in curriculum reform is the question of what mathematics should be taught. Studies centering on the question of what <u>can</u> be taught at different levels are important but do not answer this central question. Continuing consideration of content should proceed in cooperation with groups in other countries who are facing the same question. Also, growth in the use of mathematics in other areas of knowledge suggests that consideration is given in the curriculum to the mathematics which is relevant for these areas.

Standardized tests of achievement prepared by testing agencies external to schools may serve as a deterrent to the introduction of new courses. It is important that such agencies continue to seek advice from broadly representative groups, so that they may introduce such new content as is appropriate, but will not penalize students whose teachers are not yet skilful in teaching new courses.

To effect widespread improvements in curriculum, it is essential that means be found to inform teachers and administrators about the changes recommended. At present, the report of the Commission on Mathematics of the College Entrance Examination Board, of the program of the School Mathematics Study Group, the University of Illinois Committee on Secondary Mathematics,

and other groups are known mainly in urban regions. Furthermore, many college and university faculty members are unfamiliar with these programs. The admirable regional conference for administrators sponsored by the National Council of Teachers of Mathematics in 1960 reached mainly city school administrators; additional conferences to reach non-urban regions are needed. The Mathematical Association of America Visiting Lecturer Program for high school students might be expanded to inform teachers and administrators about proposed curriculum changes. Contact with the Educational Policies Commission of the National Education Association might be helpful.

A crucial problem in curriculum change is the preparation of teachers to teach the courses designed by SMSG and other groups. High school and elementary teachers with inadequate preparation approach new materials timidly and should be provided with inservice training and the assistance of consultants.

Many teachers look to mathematicians for guidance in planning the curriculum for their schools, and some are confused by apparent disagreements among mathematicians, and by differences among experimental programs. Careful examination reveals substantial areas of agreement among the experimental courses and between these and the courses recommended by the Commission on Mathematics of the C.E.E.B.

The recommendations of this Commission on course content in secondary schools have received wide approval, but not everyone approves them completely. Furthermore, recommendations alone do not produce desired changes. To accelerate change, SMSG has produced teaching materials. The volume of these is such that teachers find it difficult to see exactly what they contain. A digest of the program for grades 7 to 12, perhaps 100 pages in length, would be helpful in this connection, and similar digests of other experimental programs might well be prepared.

The SMSG materials were prepared in experimental form to serve until commercial texts by private authors incorporating the same ideas are available. While the experimental programs have affected revisions of commercial texts,

it is at present uncertain whether commercial publishing will supply satisfactory texts in the near future, unless some central stimulation or encouragement is provided. Perhaps some group should survey the SMSG program to determine whether these texts should be published in more permanent form. Also, critical reviewing of commercial texts should be encouraged.

There is need for teaching materials which will answer the students' questions about the applications of mathematics and about the relevance of mathematics to their present and future activities. Furthermore, it would be helpful to teachers to have assistance from mathematicians in answering students' questions stimulated by new teaching materials. The M.A.A. might be requested to explore the problem of supplying such assistance locally when this is possible.

The widespread interest in teaching machines suggests that attention be given to the potentialities of these for improving the teaching of mathematics. To date, the work done has reflected concern with technique, rather than with the content presented. E.T.S. has a grant to evaluate the programming of courses, and the SMSG Advisory Committee is interested in the development of sound programs for use with these machines.

The provision of adequate materials for teaching a revised curriculum does not alone produce changes in the classroom. Serious concern is felt about the problem of teacher education, both preservice and inservice. Many newly trained teachers do not have mathematical training appropriate for teaching the new courses, and the deficiencies of teachers not recently trained are more severe. Local schools introducing these courses should supply assistance for supervisors and for teachers by providing inservice training and mathematics consultants; and should encourage attendance at conferences and institutes provided by other agencies. The cooperation of the mathematical community in supplying such assistance is essential to effective use of the teaching materials which are available.

It is observed that when effective guidance is provided, the use in the

classroom of SMSG or other experimental programs is in itself a valuable means of teacher education. This is not apt to be the case without suitable guidance. At present about 3% of the nation's students are using some experimental program.

A strong need was expressed for a continuing body representative of the mathematics community to exert leadership in continuing revision of the curriculum, to encourage the production of sound teaching materials, to promote teacher education adequate for teaching the revised curriculum, to cooperate with similar groups in other countries, and to aid in interpreting the need for sound mathematical education to local school officials and the lay public. It was proposed that such a body might be established as a continuation of the SMSG Advisory Committee, by providing for rotating membership, making it responsible in some way to the Conference Board of the Mathematical Sciences, broadening its areas of activity, and finding the necessary financial support.

FINAL RECOMMENDATIONS OF THE CONFERENCE

The recommendations which follow were adopted unanimously by the Conference participants. These recommendations result from discussions and considerations following the major addresses, ensuing discussions, and the presentation of a preliminary report by the resolutions committee. The Resolutions Committee consisted of C. L. Corcoran, Roy Dubisch, A. M. Gleason, Mildred Keiffer, R. A. Rosenbaum, and Mina Rees, chairman.

INTRODUCTION AND GENERAL RECOMMENDATIONS

The SMSG has attained most of its original goals. A large number of sample texts for high schools have been prepared, they have been tested in the classroom, and are already causing major changes in the teaching of mathematics. However, it is important to continue the work in several areas not envisaged when SMSG was founded. There is a considerable demand for a modernized curriculum for the elementary school reaching all the way back to the primary grades. A continued surveillance of mathematical teaching is necessary to guarantee that our brave start shall not dwindle away. The advent of the teaching machine has obliged the SMSG to begin serious study of the place of this new device in the teaching of mathematics. A basic objective of the revised curriculum has been to teach an understanding of the subject which will affect the students long after they leave school; it is vital that long-term follow-up studies be made to find out whether this objective has been achieved.

But the problem is even larger. The fundamental objective of the whole educational process has been, and must always continue to be, to provide the best education possible for every student. As knowledge expands, this expansion must be reflected in the schools. Hence the rapid evolution of all scientific discipline and the introduction of new mathematical techniques into many hitherto largely descriptive areas must be counterbalanced by more frequent revision of the science and mathematics curricula. The most profound effects will be felt at the university level, but not even the arithmetic of the first grade can remain unchanged. There is a need therefore for a continuing body which will identify and initiate appropriate curricular changes as rapidly as possible.

Therefore, this conference proposes that:

The SMSG should continue its work in writing sample textbooks and supplementary monographs as they are needed, in preparing programs for teaching machines if feasible, and in testing the effects of its program. Moreover, the SMSG should broaden the

scope of its operations to provide leadership in all areas of school mathematics education.

Since it appears that the SMSG should broaden its scope and continue its operation far beyond the period originally imagined, it is desirable to reorganize its governing board to distribute responsibility more widely in the mathematical community. Therefore, this conference proposes that:

1. The Advisory Board of the School Mathematics Study Group shall consist of the Director and 24 members representing all areas of the country and all segments of the community concerned with mathematical education.
2. The term of service of each member shall be three years, and no member shall serve consecutive terms.
3. Each year four new members shall be appointed by the Conference Board of the Mathematical Sciences and four new members shall be elected by the Advisory Board to serve from the following September first. Vacancies occurring shall be filled by the Advisory Board.

It is suggested that this plan become effective September 1, 1961; that the present Advisory Board should select, by lot or agreement, eight of its members to serve until 1963, and eight to serve until 1962; and that eight new members be selected in accordance with (3) above.

RESEARCH AND THE DEVELOPMENT OF CURRICULA

The primary goals of the SMSG should be to foster research and development in the fields of curricular content and mathematics teaching and to take whatever steps it can to promote the widespread adoption of established advances in either course content or pedagogy. It should encourage, advise and/or cooperate with all groups which are sincerely devoted to these goals. Every effort should be made to provide the best possible education in mathematics to students at every level of ability. Specifically:

1. The SMSG should continually review the entire mathematics curriculum in the schools, with the aim of recommending in the light of

increasing experience, changes in sequence or emphasis; and it should frequently reassess the feasibility and desirability of introducing topics not now included (e.g. calculus). In order to do this effectively, it should maintain liaison with other scientific groups and their curriculum development projects.

2. A project for the development of new curricula and sample texts for the elementary schools is urgently needed.
3. Adequate and realistic programs in mathematics for all students are needed. Immediate attention should be given to the development and utilization of materials for students of ability levels lower than the college capable. This represents a major responsibility which the mathematical community cannot abdicate. Similar attention should be given to programs for mathematically gifted students.
4. Successful curricular changes must be based on research. Basic research in this area takes the form of bold experimentation which differs sharply from present practice. Experiments in education cannot be entered into lightly, but SMSG should encourage and cooperate with pilot ventures that are imaginatively planned whenever they are conducted by enthusiastic and competent people and designed so that a real contribution to our knowledge of new substance and techniques can be expected. It is only on knowledge gained from such experiments that future curricula can be planned.

IMPLEMENTATION

No program of curricular improvement can be effective unless vigorous steps are taken to promote it. The most important of these steps must be directed toward the school teachers, but we must not overlook the other segments of the community interested in our schools. Thus:

1. Every effort should be made to help teacher training institutions to develop a suitable program such as that recommended by the Panel on Teacher Training of the Committee on the Undergraduate Program in Mathematics. It is imperative that teachers be

prepared to handle, with a fair degree of ease, such new material as that prepared by SMSG, UICSM, and similar groups, and to incorporate effective teaching methods into their handling of these materials.

2. Inservice training is still needed at all levels. The present program of summer institutes, academic year institutes, and other types of inservice training should be extended and strengthened. The SMSG should explore other ways to provide inservice training. Special attention should be given to providing supervisors, both state and local, the training they need.

   The extension of curricular revision into the elementary schools underlines the importance of increasing inservice training opportunities for both teachers and supervisors at this level.

   All such programs should be carefully evaluated to prevent the waste of time and money.

3. To assist teachers who must work in a traditional framework with traditional textbooks because of local conditions or a lack of confidence in their ability to launch a whole new program the SMSG should prepare individual units which can be inserted into standard programs when the teacher sees and opportunity.

   The cooperative development of units which accurately reflect current developments and applications of mathematics will help to maintain the rapport which SMSG has established between teachers and mathematical scholars.

4. In presenting unfamiliar material, teachers can often profit greatly from even brief contacts with more knowledgeable persons. The SMSG should try to facilitate contacts between teachers and others whose specialized training or experience can be useful.

5. School teachers are constantly being asked for examples for the application of various aspects of the curriculum. SMSG should publish articles, or source books, from time to time, which

illustrate ways in which mathematics is used by scientists, engineers, and others.

6. To provide for the student who is able to do more than the regular class requirements and to help teachers maintain their orientation toward mathematics, the SMSG should continue to publish monographs on special topics in mathematics.
7. Considerable publicity must be given to changes in the mathematical curriculum. SMSG should arrange for the dissemination of information about the content and objectives of the new programs to school teachers, supervisors, school administrators, school board members, college and university teachers, parents and other members of the public. The SMSG should consider holding conferences for administrators and school board members to supplement the regional conferences begun by the NCTM. Individuals should be trained to make clear expositions of the nature of the changes in school mathematics to parents, teachers and administrators. To reach remote areas, the use of mass media should be considered.
8. The new texts themselves are such an imposing collection of books as to discourage any casual study. A booklet of perhaps 100 pages presenting the objectives of the new curricula as well as a summary of the content would be of great help to teachers, supervisors and administrators. Another pamphlet with greater emphasis on the importance of mathematics in the world today and on the justification of the recent changes should be prepared for parents.
9. Consideration should be given to defining the role of SMSG in textbook publishing. In particular, a small group of mathematicians and teachers of mathematics might profitably meet with a group of publishers to discuss problems of publishing textbooks reflecting the work of the experimental groups.

ANALYSIS AND FOLLOW-UP

As a long range study group, SMSG must continually analyse its work to determine how to improve its operations and whether its objectives are being

achieved. Some suggestions are given here:

1. SMSG might profit from a careful comparison between the present curricular projects and earlier ones. Such a study may reveal the factors which contribute significantly to a successful program. The following elements may be relevant: Adequate support, the development of text materials, experimental teaching, a favorable climate of public opinion, cooperative relations with testing agencies, continuing programs for getting relevant information to teachers and supervisors, cooperation between school teachers and mathematicians.
2. In addition to defining the general objectives of curriculum projects, considerable effort must still be made in defining operationally the criteria which reflect the objectives (other than mathematical knowledge and skills) of the mathematics programs. Teachers, pupils, and parents react strongly to evaluative instruments (e.g. college board examinations); hence, SMSG should support efforts to construct tests which reflect all of the many objectives of the programs.
3. Long-range follow-up studies, particularly of students who go on to college, should be made to find out if the training provided by the new programs meets the demands made upon it and, moreover, whether it produces the hoped for results concerning attitudes towards mathematics.
4. SMSG should encourage joint research between mathematicians and behavioral scientists concerning the learning process and the formation of attitudes towards mathematics.

CONCLUDING STATEMENT

There will be a continuing necessity to obtain adequate financial support. Very large sums of money will be needed, and the Federal Government will have to provide bulk of this money. But the support of private foundations should be sought for projects for which federal funds may not be appropriate.

Submitted on behalf of the Conference by:

Mina Rees, Chairman
The Resolutions Committee

APPENDIX A

CONFERENCE PARTICIPANTS

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APPENDIX B

Bylaws of the School Mathematics Study Group

(These bylaws resulted from recommendations of the Conference on Future Responsibilities for School Mathematics, February 24-25, 1961. The bylaws were pted by the Advisory Board of SMSG in October 1961.)

I. Purposes

The primary purpose of the SMSG is to foster research and development in the teaching of school mathematics. This research will consist, in part, of a continual review of the mathematics curricula in the schools as an aid in the selection and design of promising experiments. It will also consist, in part, of an analysis of the results of experimental teaching as an aid in judging whether the objectives of various programs are being achieved. But the work of the SMSG should consist primarily in the development of courses, teaching materials and teaching methods.

A great variety of these are needed. In the first place, the range of ability among the students who ought to be learning mathematics is so wide that special provisions need to be made so that the students at various ability levels can be taught in appropriate styles and at appropriate paces. Moreover, there should be bold experiments with courses differing sharply from present practice in their style or their content, or both; if experimentation is limited to programs whose desirability is obvious and whose feasibility is predictable then some of the best opportunities are likely to be missed. Sources of other suggestions for needed activities and projects are: (a) the declaration of aims and purposes of the SMSG, July 1958; (b) the actual activities and productions of the SMSG, 1958-1961; (c) the Recommendations of the Conference on Future Responsibilities for School Mathematics, February 1961.

It is a part of SMSG's task, in cooperation with the several mathematics organizations, to encourage exploration of the hypotheses underlying mathematics education. Care should be taken to give attention to the needs of

other disciplines interested in the mathematical preparation of students. It is also part of the SMSG's task to publicize itw own work and make this work available to people who might use it. It should be understood, however, that the basic job of the SMSG is not to defend any orthodoxy, old or new, by passing judgment on questions of educational policy but rather to make contributions to the data on which such judgments must be based. In this spirit, efforts should be made to help teacher-education institutions to offer programs which will enable their graduates to teach the new courses in the schools. The SMSG should also explore the possibility of promoting the developments of inservice education of teachers and supervisors as a continuing and necessary part of their professional lives.

The success of the SMSG/s enterprise depends on the full participation of mathematicians from the colleges and universities and classroom teachers from the schools. It is a fundamental policy that the work of the SMSG be collaborative in this sense.

II. Organization

The executive officer of the SMSG shall be the Director. There shall also be an Advisory Board and an Executive Committee.

The first Advisory Board of the SMSG shall consist of

(1) The Director, ex officio,

(2) Eight members of the SMSG Advisory Committee (as constituted in the spring of 1961), to serve for two years,

(3) Eight members of the same body, to serve for one year,

(4) Four members to be elected by the same body in the fall of 1961, to serve for three years, and

(5) Four members to be elected by the Conference Board of the Mathematical Sciences, to serve for three years.

Thereafter the term of each member shall be three years, beginning on September 1 and ending on August 31; and no member shall serve two consecutive terms.

Each year four new members shall be appointed by the Conference Board of the Mathematical Sciences and four shall be elected by the Advisory Board.

Vacancies due to uncompleted terms shall be filled by the Executive Committee.

The Advisory Board shall elect (1) its own chairman, who shall also be the chairman of the Executive Committee, and (2) three other members of the Executive Committee. All of these officers shall serve for one-year terms, beginning on September 1 and ending on August 31.

The Executive Committee shall consist of its four elected members together with the Director, ex officio. If the directorship becomes vacant a new Director shall be appointed by the Executive Committee.

The chairman of the Advisory Board shall preside at all meetings of the Advisory Board and the Executive Committee. Each year he shall appoint a nominating committee which shall make nominations for all of the annual elections by the Advisory Board. At the meeting at which these elections are held nominations from the floor shall be permitted.

Meetings of the Advisory Board and the Executive Committee shall be called by the Director. The Director shall also appoint such ad hoc committees, panels and writing teams as may be needed, and prepare reports on SMSG activities on request of the Advisory Board.

Amendments to the Bylaws may be made by the Advisory Board.